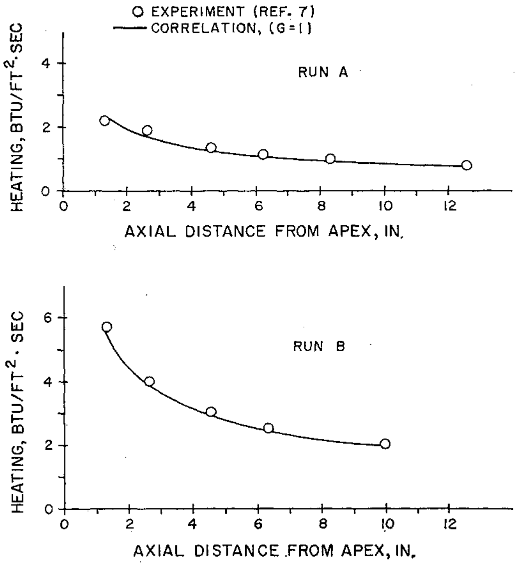

Fig. 1.

nation point and is essentially independent of the value of $\rho_e\mu_e/\rho_w\mu_w$ for the two bodies. In light of this result, then, one obtains G from the correlation given in Fig. 1 of Ref. 3:

$$(g_c'/g_s')_w = G = [1 + 0.096(\tfrac{1}{2})^{1/2}]^{-1} = 0.936 \quad (12)$$

However, upon closer examination, Fig. 1 of Ref. 3 shows that the fluid property plus dissipation term modifications move the points lying on the $\beta = 0$ curve sufficiently upward and to the right such that, in comparing the stagnation ($U_e^2/H_e = 0$) to the cone ($U_e^2/H_e \rightarrow 2$) solutions, one is quite justified in taking G equal to 1. In fact, it will be seen that the few experimental data available imply a value for G slightly greater than one.

Recently, Hanley[5] published a correlation of Cohen's numerical results[6] for laminar heating to flat plates at high speed in the form

$$q = 1.17(10^4)\left(\frac{1}{x}\frac{U_e}{U_\infty}\frac{p_e}{p_\infty}\rho_\infty\right)^{1/2}\left(\frac{T_w}{900}\right)^{-0.051} \times \left(\frac{U_\infty}{10^4}\right)^{3.21}\left(1 - 1.205\frac{h_w}{H_e}\right) \quad (13)$$

Since heating to cones is greater by a factor $3^{1/2}$ than that to flat plates, a correlation in Hanley's form can be obtained immediately from Eqs. (1, 5, and 9) as

$$\frac{q_c}{3^{1/2}} = 1.272(10)^4 G\left(\frac{1}{x_c'}\frac{U_c}{U_\infty}\frac{p_c}{p_s}\rho_\infty\right)^{1/2}\left(\frac{U_\infty}{10^4}\right)^{3.35} \quad (14)$$

Besides a small difference in velocity dependence of 3.21 instead of 3.35, Hanley's constant implies a value for G of 0.92, which is close to that from Kemp et al. [cf., Eq. (12)]. One concludes that the calculations of Cohen and Kemp et al. are in very good agreement if one assumes that $g_w'/(1 - g_w)$ is independent of $\rho_e\mu_e/\rho_w\mu_w$.

The scarcity of heating data for cones in hypersonic flight at zero angle of attack allows only tentative confirmation of the theory. Figure 1, using the data of Ref. 7, shows two runs made at the conditions shown in Table 1.

The data are seen to be correlated very well by Eq. (10) with $G = 1$, although the consistently lower heating predicted by this relation indicates that a value of G slightly greater than 1 perhaps may be even better. However, no definite conclusions can be drawn until more data are available, and so using $G = 1$ should give accurate predictions of laminar heating to cones.

Table 1 Test conditions

Run	M_∞	H_∞, ft²/sec²	$\rho_\infty U_\infty^2/2$, psi	ρ_∞/ρ_{sl}	δ
A	15.89	27.3(10⁶)	0.094	2.09(10⁻⁴)	8.3°
B	16.77	28.7(10⁶)	0.435	9.2(10⁻⁴)	8.3°

The variable ξ_c [cf., Eq. (4)] was determined under the assumption of a pointed cone, whereas in reality re-entering cones will become blunted to some extent. Lees[8] has shown, however, that blunting has a minor effect on the conical heating when the length is measured from the virtual apex of the blunted cone. This follows from the assumption of local similarity, which neglects flow history up to any point, and from the fact that the pressure on the conical portion is the same (within the accuracy of the Newtonian approximation) whether a cone is blunted or not. Hence, Eq. (10) (along with the wall enthalpy correction) can be used over any laminar portion of a trajectory with $G = 1$.

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Approximate Analysis of Propellant Stratification

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Introduction

STRATIFICATION refers to the development of temperature gradients in a fluid container subjected to external heating. A free-convection boundary layer carries the heated fluid at the walls to the top of the tank, forming a growing layer of "stratified" liquid which is at a higher temperature than the bulk. Thus, ullage pressure rise of a saturated fluid in a closed container is greater than the pressure rise that would occur if the fluid were mixed at a uniform temperature.

The system analyzed is a closed cylindrical tank accelerating along its longitudinal axis and filled to some height H with liquid, which is subjected to a uniform side wall heat flux $\dot{q}$. The ullage is assumed to be at the saturation pressure corresponding to the liquid surface temperature. Wall heat flux to the ullage is considered to be negligible. At time zero, a steady turbulent wall boundary layer is formed instantaneously in the liquid, which is at a constant bulk temperature T_b. The unknowns of the problem are the surface temperature T_s, the ullage pressure P_u, and the stratified layer thickness Δ.

An approximate solution is obtained by assuming a di-

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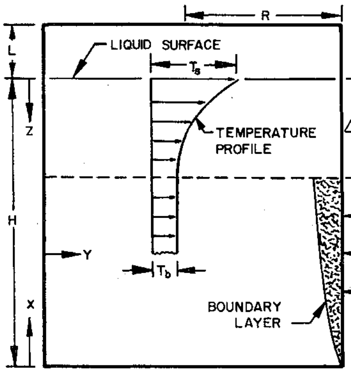

Fig. 1 Geometry used in stratification analysis.

mensionless temperature profile in the stratified layer, the growth of which is determined by a simple mass balance.

Mass Balance

The growth of the stratified layer is governed by the rate at which fluid in the boundary layer crosses the plane $z = \Delta(t)$. The mass flow in the boundary layer is

$$\dot{m}_{bl} = 2\pi R\rho \int_0^\delta u\, dy \qquad (1)$$

where ρ is the liquid density, u is the local velocity in the boundary layer, and δ represents the boundary layer thickness. Geometric terms are shown in Fig. 1.

A solution for a turbulent free convection boundary layer with constant wall heat flux $\dot{q}$ and uniform bulk temperature has been obtained by Morse[1] using the velocity and temperature profiles employed by Eckert and Jackson[2] in their analyses of turbulent free convection at constant wall temperature. Using Morse's results, Eq. (1) can be written as

$$\dot{m}_{bl} = 2\pi R\rho[0.286\ \nu^{1/7}(g\beta\dot{q}/\rho c)^{2/7}\ x^{8/7}] \qquad (2)$$

In this relation ν, c, and β are the kinematic viscosity, specific heat, and expansion coefficient of the liquid, respectively, and g is the acceleration of the system. The symbol x is the boundary layer run length.

The rate at which mass increases in the stratified layer is

$$\dot{m}_{sl} = \pi R^2\rho(d\Delta/dt) \qquad (3)$$

Equating (2) and (3) and letting $x = H - \Delta(t)$ yields the following differential equation for $\Delta(t)$:

$$d\Delta/dt = (C_T/R)[\dot{q}(g/g_0)]^{2/7}\ (H - \Delta)^{8/7} \qquad (4)$$

where

$$C_T = 0.573\ \nu^{1/7}(g_0\beta/\rho c)^{2/7} \qquad (5)$$

and g_0 is used to represent standard acceleration.

Integration of (4) gives the following result for the growth of the stratified layer:

$$\Delta/H = 1 - [1 + 0.082(H/R)(Gr^*/Pr)^{2/7}\phi]^{-7} \qquad (6)$$

where $Gr^* = g\beta\dot{q}H^4/k\nu^2$, and $\phi = \nu t/H^2$; k is the thermal conductivity and Pr the Prandtl Number.

The stratified layer growth due to laminar free convection boundary layers is obtained in a similar manner. Using Sparrow's results[3] for laminar free convection, one obtains

$$\frac{\Delta}{H} = 1 - \left[1 - 0.616\,\frac{H}{R}\,\frac{(Gr^*)^{1/5}\,\phi}{(\frac{4}{5} + Pr)^{1/5}\,Pr^{3/5}}\right]^5 \qquad (7)$$

Energy Integral

The energy entering the tank increases the temperature of the growing stratified layer and vaporizes mass from the liquid surface to maintain an equilibrium condition in the ullage region. (Although the mass transfer to the ullage is negligible, the energy transfer is not.) The temperature distribution in the liquid is considered to be one-dimensional, and the tank diameter is considered to be large in comparison with the boundary-layer thickness. An energy balance for this situation yields

$$2\pi RH \int_0^t \dot{q}dt = \pi R^2\rho c \int_0^\Delta (T - T_b)dz + Q_v \qquad (8)$$

where T is the local temperature at any point in the stratified layer, T_b is the bulk temperature, and Q_v is the heat used in vaporizing the liquid. If the rate of change of density with temperature under saturation conditions $(\partial\rho/\partial T)_s$ is considered to be constant in the range of interest, Q_v is given by

$$Q_v = L_v(\partial\rho/\partial T)_s\,(T_s - T_b)V_u \qquad (9)$$

where L_v is the average latent heat over the range of interest and V_u is the ullage volume πR^2L

An energy integral I is now introduced:

$$I = \int_0^1 \theta d\eta \qquad (10)$$

where θ is a dimensionless temperature rise and η is a dimensionless stratified layer thickness given by

$$\theta = (T - T_b)/(T_s - T_b) \qquad \eta = z/\Delta \qquad (11)$$

The energy integral represents the average temperature rise in the stratified layer normalized with respect to the surface temperature rise. If it is assumed that the dimensionless temperature profiles in the stratified layer remain invarient with time, the surface temperature may be computed from Eqs. (8) and (9):

$$T_s - T_b = \frac{2H\dot{q}t}{\rho CRI\Delta + L_v(\partial\rho/\partial T)_sRL} \qquad (12)$$

where $\Delta(t)$ and I are given by Eqs. (6) or (7) and (10), respectively.

Experimental Results

Stratification tests were conducted with water in a cylindrical container ($H = 11.3$ in., $R = 4.9$ in.) at five values of heat flux ranging from 0.71 to 3.7 Btu/ft²-sec. Temperature measurements were made at 12 locations along the tank centerline to determine temperature profiles and observe the growth of the stratified layer. The magnitude of $(Gr^*\,Pr)$ in these tests was such that transition from laminar to turbulent flow occurred part way up the wall in each test.

The experimental values of Δ/H for all five tests were between the values predicted by Eqs. (6) and (7) but agreed best with the turbulent prediction. Figure 2 compares the measured growth of the stratified layer with the turbulent expression. The data represent test times ranging between 60 and 260 sec.

The temperature measurements from each test were reduced to dimensionless form. It was possible to fit the data with a curve of the form $\theta = (1 - \eta)^n$ for each test. The

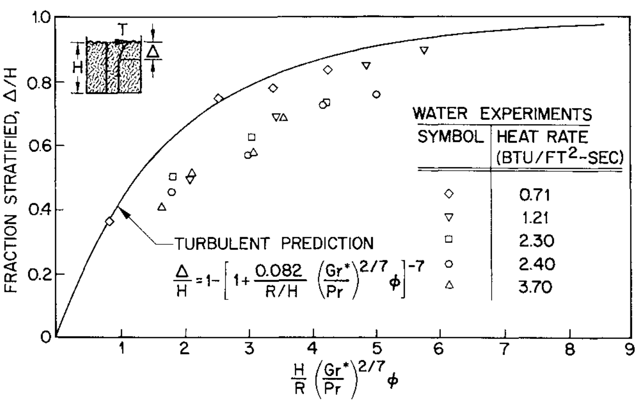

Fig. 2 Comparison of observed and predicted stratified layer growth.

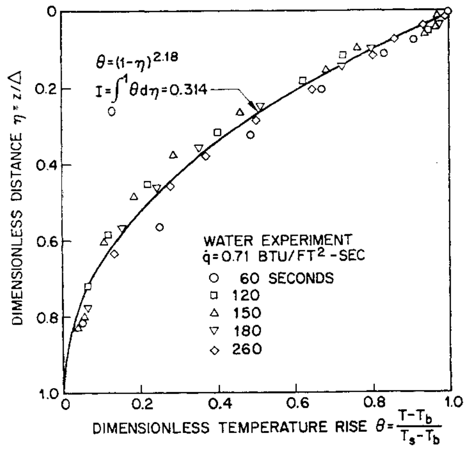

Fig. 3 Typical dimensionless temperature profiles.

values of n ranged from 1.48 to 2.18, resulting in energy integrals I between 0.31 and 0.40. The data shown in Fig. 3 are typical of the nondimensional profiles obtained.

Figure 4 compares the predicted nondimensional temperature rise with the experimental results. The turbulent expression for the stratified layer growth, Eq. (6), was used together with a value of 0.33 for the energy integral. The prediction was made without the evaporation term in the energy equation, since each test was terminated before the liquid surface reached the saturation temperature.

Conclusions

An approximate analysis of the transient stratification of a closed fluid container subjected to constant wall heating has been obtained. The results rely upon the assumption that the dimensionless temperature profiles in the stratified layer do not vary with time. The solution also is restricted to times when the stratified layer thickness is somewhat less than the liquid height. Preliminary experimental data tend to corroborate the analytic results. Further experiments are needed to determine whether or not the dimensionless temperature profiles are always time-invarient, and, if they are, how they are affected by tank size and fluid properties. Until such data are available, the results presented must be considered as tentative; nonetheless, they are useful in pointing out the significant physical and geometric variables governing the phenomenon.

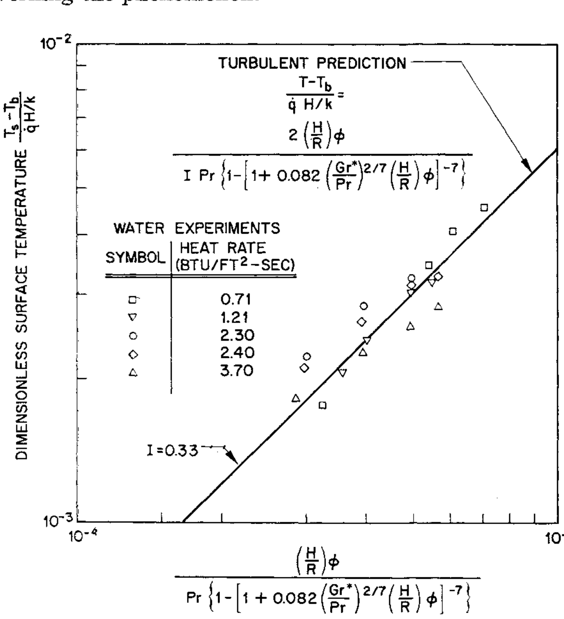

Fig. 4 Comparison of observed and predicted surface temperature rise.

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Conduction in Thin-Skinned Heat Transfer and Recovery Temperature Models

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Nomenclature

c = mass specific heat of skin material
E = fractional error due to tangential conduction
h = heat transfer coefficient
k = thermal conductivity of skin material
L = model length
n = harmonic number
q = convective heat transfer rate per unit area
T = skin temperature
T_r = recovery or adiabatic wall temperature
T_i = initial skin temperature
T_{nc} = skin temperature solution neglecting conduction
t = time
t_1, t_2 = restrictions on minimum measuring time as defined in text
α = thermal diffusivity of skin material $= (k/\rho c)$
δ = model skin thickness
δ_0 = scale of the variable skin thickness; (δ/δ_0) is a dimensionless function of the surface coordinates
ρ = density of skin material

THIS note deals with the conduction errors involved in the thin-skinned method of measuring convective heat transfer rates and recovery temperatures. This method uses a wind tunnel model with a thin, usually metal, skin. At the start of an experiment, the model is at a steady, uniform temperature. The gas flow over the model then is established quickly, for example, by injecting the model into the gas stream, and the time variation of the skin temperature is recorded at different positions on the model. The experimenter commonly considers the temperature gradient within the skin normal to the surface to be zero, assumes conduction parallel to the surface and across the inner surface of the skin to be negligible, and thus arrives at the following simple heat balance for an element of skin:

$$h(T_r - T) = q = \rho c \delta(\partial T/\partial t) \tag{1}$$

Then, by measuring $(\partial T/\partial t)$ shortly after the beginning of the heating or by extrapolating backward from later time, he obtains the isothermal heating rate q. Finally, the skin temperature will approach asymptotically its adiabatic or recovery value T_r, if the conduction is again negligible. With q and T_r known, the heat transfer coefficient h may be com-

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